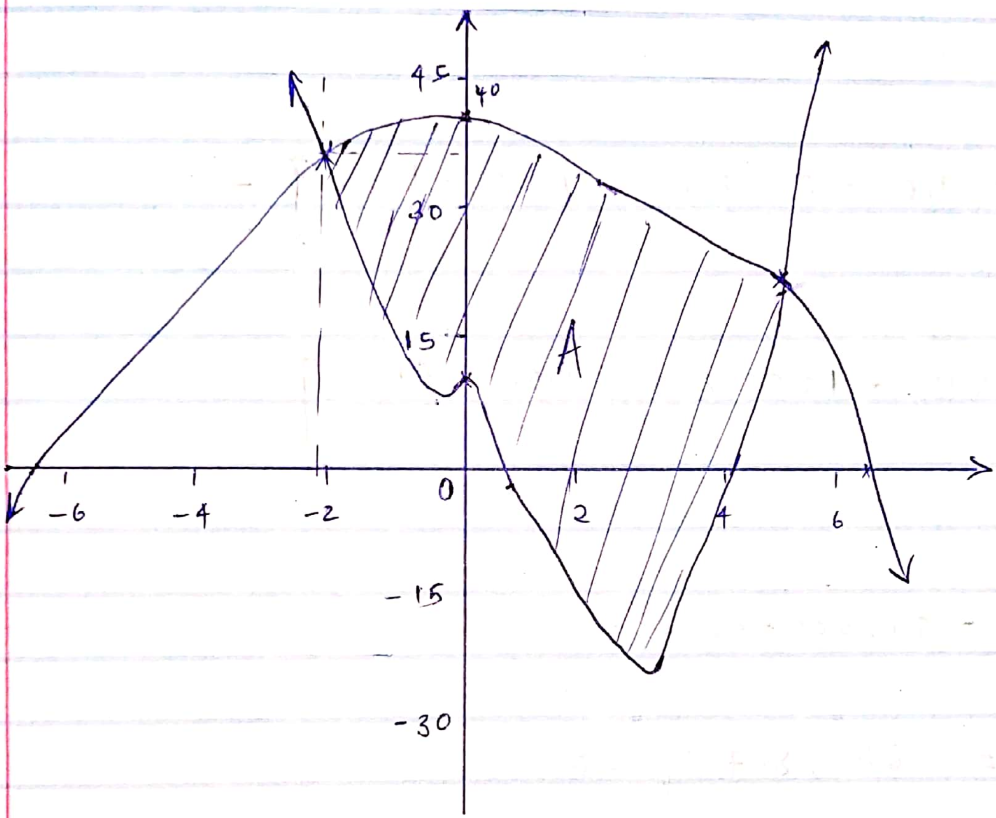
Question 2

$y = x^4 - 3x^3 - 4x^2 + 10$ and $y = 40 - x^2$



$x^4 - 3x^3 - 4x^2 + 10 = 40 - x^2$

$x^4 - 3x^3 - 3x^2 + 10 = 40$; $x^4 - 3x^3 - 3x^2 - 30 = 0$

Using Newton-Rhapson method $x \approx -2.0349$, $x \approx 4.145$

$y(-2.0349) = 40 - (-2.0349)^2 = 35.859$

$y(4.145) = 40 - (4.145)^2 = 22.819$

$$A = \int_{-2.0349}^{4.145} (x^4 - 3x^3 - 4x^2 + 10 - 40 + x^2)\, dx$$

$$= \int_{-2.0349}^{4.145} (x^4 - 3x^3 - 3x^2 - 30)\, dx$$

$$= \left[\frac{x^5}{5} - \frac{3}{4}x^4 - x^3 - 30x \right]_{-2.0349}^{4.145}$$

$$= \left[244.7105 - 221.3907 - 71.2153 - 124.35 \right] -$$

$$\left[-6.9782 - 12.8598 + 8.4262 + 61.047 \right]$$

$$= -172.2455 - 49.6352$$

$$= 221.8807 \text{ sq units}$$

Question 3

(a) $\displaystyle\int_0^1 5x \times \sqrt{x^2+3}\, dx$

Apply U-substitution : $u = \cancel{\sqrt{x^2+3}}\ x^2+3$

$$\frac{du}{dx} = 2x$$

$$dx = \frac{du}{2x}$$

$$= 5\int_0^1 \cancel{x} \times \sqrt{u}\, \frac{du}{2\cancel{x}} \quad = \frac{5}{2}\int_0^1 \sqrt{u}\, du$$

$$\int \sqrt{u}\, du = \int u^{1/2}\, du = \frac{u^{1/2+1}}{1/2+1} = \frac{2}{3} u^{3/2}$$

Substituting back : $2/3\,(x^2+3)^{3/2}$

$$= \frac{5}{2}\left[\frac{2}{3} \times (x^2+3)^{3/2}\right]_0^1$$

$$= \frac{5}{3}\left[\sqrt{(x^2+3)^3}\right]_0^1 = 5/3\left(\sqrt{64} - \sqrt{27}\right)$$

$$= \frac{5}{3}\left(8 - 3\sqrt{3}\right)$$

$$= 5\left(\frac{8}{3} - \sqrt{3}\right)$$

(b) $$\int \frac{2x\,dx}{(1+x^2)} = 2\int \frac{x\,dx}{(1+x^2)}$$

Substituting $u = x^2+1$; $\frac{du}{dx}$ $2x$; $dx = \frac{du}{2x}$

$$= 2\int \frac{\cancel{x}\, du}{u \cdot 2\cancel{x}} = \int \frac{1}{u}\, du$$

$$= \ln|u| + c$$

Substituting back : $\ln|x^2+1| + c$

(c) $\int x e^{x^2} dx$ Substituting $u = x^2$; $\frac{du}{dx} = 2x$; $dx = \frac{du}{2x}$

$$= \int \cancel{x} e^{u} \cdot \frac{du}{2\cancel{x}}$$

$$= \frac{1}{2} \int e^{u} du \quad ; \int e^{u} du = e^{u}$$

$$= \frac{1}{2} e^{u}$$

Substituting back: $\frac{1}{2} e^{x^2} + c$

(d) $\int x \times \sqrt{5x+1} \cdot dx$

Applying U-substitution; $u = 5x+1$; $\frac{du}{dx} = 5$; $dx = \frac{du}{5}$
$x = \left(\frac{u-1}{5}\right)$

$$= \int \left(\frac{u-1}{5}\right) \times \sqrt{u} \times \frac{du}{5}$$

$$= \frac{1}{25} \int (u^{3/2} - u^{1/2}) du$$

$$= \frac{1}{25} \left[\int u^{3/2} du - \int u^{1/2} du \right]$$

$$= \frac{1}{25} \left[\frac{u^{3/2+1}}{3/2+1} - \frac{u^{1/2+1}}{1/2+1} \right]$$

$$= \frac{1}{25}\left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right]$$

$$= \frac{2}{125}u^{5/2} - \frac{2}{75}u^{3/2}$$

Substituting back: $\frac{2}{125}(5x+1)^{5/2} - \frac{2}{75}(5x+1)^{3/2}$

$$= \frac{2(5x+1)^{5/2}}{125} - \frac{2(5x+1)^{3/2}}{75} + C$$

(c) $\int \frac{dx}{x(3-x)} = -1\int \frac{dx}{x^2-3x}$

$$= -1\int \frac{dx}{(1-3/x)x^2}$$

$u = 1 - 3/x$; $\frac{du}{dx} = \frac{3}{x^2}$, $dx = \frac{x^2 \cdot du}{3}$

$$= -1\int \frac{du \cdot \cancel{x^2}}{3(u)\cancel{x^2}} = \frac{-1}{3}\int \frac{du}{u}$$

$$= -\frac{1}{3}\ln|u| + c$$

Substituting back:

$$= \frac{-1}{3}\ln\left(1 - 3/x\right) + c$$

Question 4

$$\int \frac{x}{(x-1)^3}\,dx$$

let $u = x-1$; $x = u+1$

$\frac{du}{dx} = 1$; $dx = du$

$$= \int \frac{u+1}{u^3}\,du \xrightarrow{\text{Expanding}} \int\left(\frac{1}{u^2} + \frac{1}{u^3}\right)du$$

$$= \int \frac{1}{u^2}\,du + \int \frac{1}{u^3}\,du = \int u^{-2}\,du + \int u^{-3}\,du$$

$$= \frac{u^{-2+1}}{-2+1} + \frac{u^{-3+1}}{-3+1}$$

$$= \frac{u^{-1}}{-1} + \frac{u^{-2}}{-2} \Rightarrow -1\left(\frac{1}{u} + \frac{1}{2u^2}\right).$$

Substituting back:

$$-1\left(\frac{1}{x-1} + \frac{1}{2(x-1)^2}\right) + c$$

Question 5

$$f(x) = \frac{1}{x}$$

(a) $\lim_{x \to 0^-} f(x)$

x	-0.1	-0.01	-0.001	-0.0001	-0.00001
$f(x)$	-10	-100	-1000	-10000	-100000

The smaller x gets, the bigger $\frac{1}{x}$ gets to the negative

$$\therefore \lim_{x \to 0^-} \frac{1}{x} = -\infty$$

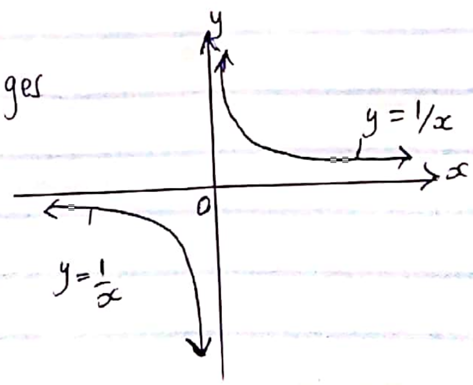
(b) $\lim_{x \to 0^+} f(x)$

(x)	0.1	0.01	0.0001	0.00001	0.00001
$(f(x))$	10	100	1000	10000	100000

The smaller x gets, the bigger $\frac{1}{x}$ gets to the positive

$$\therefore \lim_{x \to 0^+} \frac{1}{x} = +\infty$$

(c) $\lim_{x \to 0} \frac{1}{x}$ does not exist. The Left hand limit $\neq$ Right hand limit. The limit diverges



Question 6

$f(x) = x^2$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + h^2 + 2hx - x^2}{h}$$

$$= \lim_{h \to 0} \frac{h^2 + 2hx}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(h + 2x)}{\cancel{h}}$$

$= 2x$; The derivative of x^n is $n \times x^{n-1}$

$f'(x) = 2x$

$f'(3) = 2 \times 3 = 6$

$f'(4) = 2 \times 4 = 8$

(b) $f(x) = -x^3 + 6x^2$

Points at which tangent line is horizontal are local maximum and local minimum points.

$f'(x) = -3x^2 + 12x$

$f'(x) = 0$ at Maxima & minima

Thus $-3x^2 + 12x = 0$

$3x^2 = 12x$; $x = 0$

$3x = 12$

$x = 4$

$x = 0$

= at $x = 4$; $y = (-(4)^3 + 6(4)^2)$

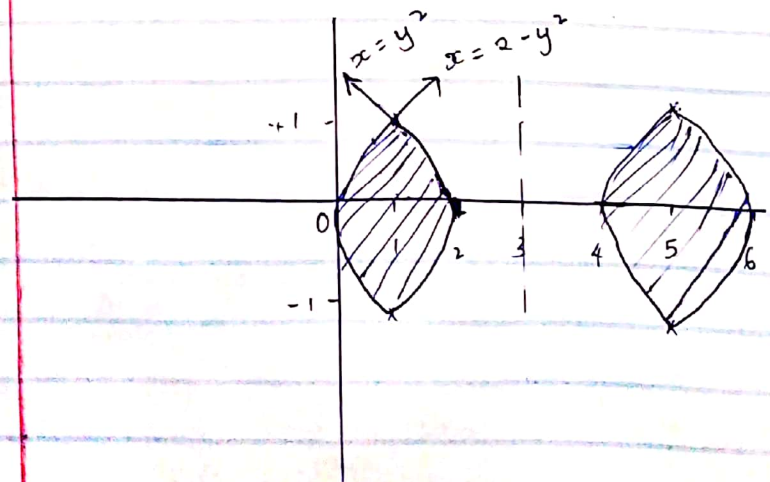
$= 32$

$y(0) = 0$

= $(4, 32)$ and $(0, 0)$

<u>Question 7</u>

$x = y^2$ and $x = 2 - y^2$ rotated about $x = 3$



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$$V = \pi \int_a^b \left([f(y)]^2 - [g(y)]^2 \right) dy$$

Shifting $\Rightarrow$ $x = y^2 - 3$

$x = -1 - y^2$

$b = 1$, $a = -1$, $f(y) = y^2 - 3$

$g(y) = (-1 - y^2)$

$$V = \pi \int_{-1}^{1} \left[(y^2-3)^2 - (-1-y^2)^2 \right] dy$$

$$= \pi \int_{-1}^{1} \left[y^4 - 6y^2 + 9 - (1 + y^4 + 2y^2) \right] dy$$

$$= \pi \int_{-1}^{1} \left[y^4 - 6y^2 - 2y^2 - y^4 + 9 - 1 \right] dy$$

$$= \pi \int_{-1}^{1} \left[-8y^2 + 8 \right] dy = \pi \left[\frac{-8}{3} y^3 + 8y \right]_{-1}^{1}$$

$$= \pi \left[16/3 + 16/3 \right]$$

$$= \frac{32\pi}{3} \text{ cubic units}$$